

Activity #5

Please put all work on a separate piece of paper!
Show all work in order to receive credit.

1. Evaluate the piecewise defined function at the indicated values, then sketch the graph of the function.

$$f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ 2x - 5 & \text{if } x > 1 \end{cases}$$

$$f(0) =$$

$$f(2) =$$

2. A hotel chain charges \$75 each night for the first two nights and \$50 for each additional night's stay. The total cost T is a function of the number of nights x that a guest stays.
- Find a piecewise function that defines $T(x)$. Show where the answer comes from!
 - Find $T(2)$, $T(3)$, and $T(5)$.
 - What does $T(5)$ represent?

GRAPH for 3-6.

3. $y = \begin{cases} 2 & \text{if } x > 0 \\ -3 & \text{if } x \leq 0 \end{cases}$ Also find: Domain, Range, $f(-2)$, $f(0)$ and $f(3)$

4. $y = \begin{cases} x + 1 & \text{if } x \geq 1 \\ 2x - 1 & \text{if } x < 1 \end{cases}$ Also find: Domain, Range, $f(-2)$, $f(1)$ and $f(3)$

5. $y = \begin{cases} x & \text{if } x > 0 \\ 3 & \text{if } x = 0 \\ x & \text{if } x < 0 \end{cases}$ Also find: Domain, Range, $f(-2)$, $f(0)$ and $f(3)$

6. $y = \begin{cases} x^2 & \text{if } x > 0 \\ x - 1 & \text{if } x \leq 0 \end{cases}$ Also find: Domain, Range, $f(-2)$, $f(0)$ and $f(3)$

Examples: Find the simplified difference quotient for each function.

7. $f(x) = x^3 - 2x$

8. $g(x) = \sqrt{x} - 1$

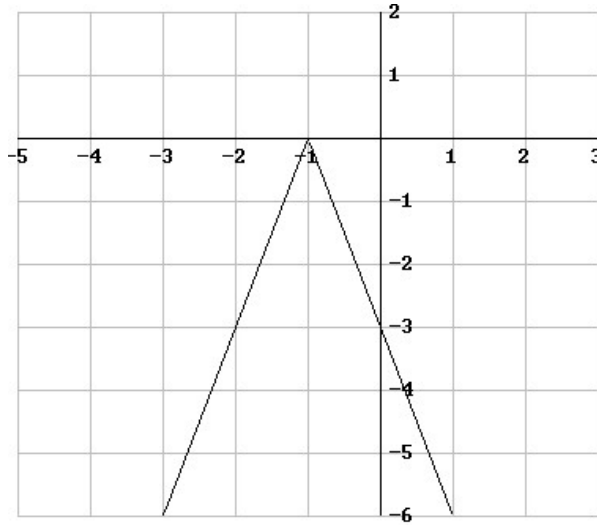
9. $h(x) = \frac{3}{2}x - 5$

10. $k(x) = \frac{1}{x-1}$
11. $j(x) = x^2 - 3x - 6$
12. Determine algebraically whether each function is even, odd or neither.
 a) $f(x) = x + |x|$ b) $f(x) = \sqrt[3]{2x^2 + 1}$ c) $f(x) = \frac{x}{x^2-1}$
13. Use a graphing utility to graph the functions below and show your graph. Find the intervals of increase and decrease. Find any local maxima and local minima. Label the graph accordingly. Round answers to two decimal places.
- a) $f(x) = x^4 - x^2$
 b) $f(x) = .2x^3 - .2x^2 - 5x - 4$
14. Find the equation of the secant line containing $(-2, f(-2))$ and $(1, f(1))$ for $f(x) = x^2 - 5$.
15. The concentration C of a medication in the bloodstream t hours after being administered is modeled by the function

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085.$$
 a) After how many hours will the concentration be highest (i.e. where is the max)?
 b) A woman nursing a child must wait until the concentration is below 0.5 before she can feed him. After taking the medication, how long must she wait before feeding her child (hint: graph $y=0.5$ and see where the concentration function lies below this line)?
16. Let $f(x) = \frac{1}{x-3}$ and $g(x) = \frac{x}{x+2}$.
- (a) Find the formula for $(g \circ f)(x)$. Simplify your formula completely.
 (b) What is the domain of $(g \circ f)(x)$?
17. Give the formula for the function whose graph has the same shape as $y = \llbracket x \rrbracket$ but stretched vertically by a factor of 3, and shifted right 1 unit.
18. The graph of the function $y = f(x)$ contains the point $(2,5)$.
 a) What point is guaranteed to be on the graph of $y = f(\frac{1}{2}x) + 3$?
 b) What point is guaranteed to be on the graph of $y = f(x+3) - 5$?
 c) What point is guaranteed to be on the graph of $y = -2f(x-2) + 1$?
 Show your reasoning.

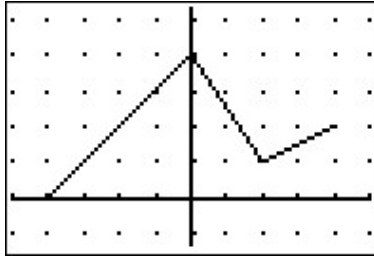
19. The graph of what function is given below?

$f(x) =$



20. How does the graph of $y = \sqrt{8 - 4x}$ compare with the graph of $y = \sqrt{x}$? (List the transformations and show where they come from.
Hint: Factor inside the radical!)
21. Suppose that the x-intercepts of the graph of $f(x)$ are -8 and 1.
- a) What are the x-intercepts of the graph of $y = f(x + 4)$?
 - b) What are the x-intercepts of the graph of $y = f(x - 3)$?
 - c) What are the x-intercepts of the graph of $y = 2f(x)$?
 - d) What are the x-intercepts of the graph of $y = f(-x)$?
- Show your reasoning (you can use a graph or words or function notations).
22. Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.
- a) Over what interval is the graph of $y = f(x + 2)$ decreasing?
 - b) Over what interval is the graph of $y = f(x - 5)$ decreasing?
 - c) What can be said about the graph of $y = -f(x)$?
 - d) What can be said about the graph of $y = f(-x)$?
- Show your reasoning (you can use a graph or words or function notations).

23. The graph of a function f is given below. Use the graph to determine the following:
Be sure to **list all the transformations** involved as well as **make a graph of the new function**.



- a) $y = f(x - 2)$ b) $y = f(x) - 2$
 c) $y = 2f(x)$ d) $y = -f(x) + 3$
 e) $y = f(-x)$ f) $y = \frac{1}{2}f(x - 1)$
 g) $y = f(2x)$
24. Find $(f \circ g)(x)$ and the domain of $(f \circ g)(x)$ for the following:
 a) $f(x) = \sqrt{x}$, $g(x) = 2 - 3x$
 b) $f(x) = \sqrt{x + 5}$, $g(x) = x^2 - 5$
25. Find $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$.

$$h(x) = \sqrt{\frac{x-5}{x+2}}$$
26. Let $f(x) = \frac{4}{x+1}$ and $g(x) = \frac{1}{6-x}$, find:
 a) $(f \cdot g)(x)$
 b) $(f + g)(x)$ and the domain of $(f + g)(x)$
 c) $(\frac{f}{g})(x)$ and the domain of $(\frac{f}{g})(x)$
 d) $(f - g)(3)$

Answers:

1. $f(0) = 3, f(2) = -1$
2. a) $\begin{cases} 75x & 0 \leq x \leq 2 \\ 50x + 50 & x > 2 \end{cases}$ b) $T(2) = 150, T(3) = 200, T(5) = 300$
- c) It costs \$300 for a 5 night stay.
3. $f(-2) = -3, f(0) = -3, f(3) = 2, D : (-\infty, \infty), R : \{-3, 2\}$
4. $f(-2) = -5, f(1) = 2, f(3) = 4, D : (-\infty, \infty), R : (-\infty, 1) \cup [2, \infty)$
5. $f(-2) = -2, f(0) = 3, f(3) = 3, D : (-\infty, \infty), R : (-\infty, 0) \cup (0, \infty)$
6. $f(-2) = -3, f(0) = -1, f(3) = 9, D : (-\infty, \infty), R : (-\infty, -1] \cup (0, \infty)$
7. $3x^2 + 3xh + h^2 - 2$
8. $\frac{1}{\sqrt{x+h} + \sqrt{x}}$
9. $\frac{3}{2}$
10. $\frac{-1}{(x+h-1)(x-1)}$
11. $2x + h - 3$
12. a) neither b) even c) odd
- 13a. Local Max: (0,0), Local Min: ($\pm .71, -.25$),
Increasing: $(-.71, 0) \cup (.71, \infty)$, Decreasing: $(-\infty, -.71) \cup (0, .71)$
- 13b. Local Max: $(-2.57, 4.13)$, Local Min: $(3.24, -15.50)$,
Increasing: $(-\infty, -2.57) \cup (3.24, \infty)$, Decreasing: $(-2.57, 3.24)$
14. $y = -x - 3$
15. a) 2.16 hours b) Nurse either before .71 hours or after 4.47 hours
16. a) $\frac{1}{2x-5}$ b) $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, 3) \cup (3, \infty)$
17. $y = 3\text{int}(x - 1)$
18. a) (4,8) b) (-1,0) c) (4,-9)
19. $f(x) = -3|x + 1|$
20. Vertical Stretch by a factor of 2 (or horizontal compression by $\frac{1}{4}$)
Horizontal Reflection over y-axis
Right 2
21. a) $(-12, 0), (-3, 0)$ b) $(-5, 0), (4, 0)$ c) $(-8, 0), (1, 0)$ d) $(8, 0), (-1, 0)$
22. a) $(-4, 5)$ b) $(3, 12)$ c) increases on $(-2, 7)$ d) increases on $(-7, 2)$
23. a) right 2 b) down 2 c) vertical stretch by 2 d) Reflect over x-axis, up 3
e) Reflect over y-axis, f) Vertical compression by $\frac{1}{2}$, right 1
- 24a. $(f \circ g)(x) = \sqrt{2 - 3x}, (-\infty, \frac{2}{3}]$
- 24b. $(f \circ g)(x) = |x|, (-\infty, \infty)$
25. $f(x) = \sqrt{x}, g(x) = \frac{x-5}{x+2}$ (answers may vary)
- 26a. $\frac{4}{(x+1)(6-x)}$
- 26b. $\frac{4}{x+1} + \frac{1}{6-x}, (-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 26c. $\frac{4(6-x)}{x+1}, (-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 26d. $\frac{2}{3}$